A Statistic Method for Amplitude Measurements

Geert Langereis
Department of Engineering Physics
Fontys University of Applied Sciences
Eindhoven, The Netherlands
g.langereis@fontys.nl

André Dommels
Department of Engineering Physics
Fontys University of Applied Sciences
Eindhoven, The Netherlands
a.dommels@fontys.nl

Jo Theunissen
Department of Mathematics and Computer Science
Technical University of Eindhoven
Eindhoven, The Netherlands
j.g.a.theunissen@tue.nl

***Abstract*—A method is proposed to detect the maximum value of a sine wave by randomized phase sampling with subsequent software processing. The method is simple, meaning it does not require additional hardware, and works over many decades of frequencies which is not the case with conventional electronic implementations.**

Keywords—Maximum detection, envelope detection, impedance measurements

I. Introduction

Impedance spectroscopy is the method to derive equivalent electronic system components from electric impedance measurements. It is commonly used in electrochemistry [1, 2] to derive surface and bulk properties of electrode-electrolyte systems. The same technique, although not always named as such, is used in specific applications to determine equivalent model components. Many sensors are described in terms of equivalent electronic components [3]. Characterization of sensor systems and transducers in general can be done based on electronic lumped element models comprising the basic elements of resistors, inductors and capacitors. This technique was first described in depth for acoustic transducers (speakers and microphones) by Beranek in 1954 [4].

With impedance spectroscopy, various AC-signals of different frequencies are imposed on a system, while the voltage-current amplitude ratio (magnitude) and phase shift are measured. The amplitude and phase signals are the indicators for the complex impedance of the system. The complex impedance as a function of frequency can be plotted in either a polar plot or a Bode diagram. The common technique of impedance spectroscopy is based on fitting asymptotes in the Bode diagram or semicircles in the polar plot. The fitted results give the circuit elements both in network-shape as in value. The original method was developed mainly by Macdonald [5] in 1987 and refined by Boukamp in 1995 [6].

From the complex function theory, we know that under certain circumstances there is a deterministic relation between the imaginary and real data (or magnitude and phase) in a single spectrum. Mathematically, the condition for this relation is a causal system. Practically it means the system is passive and stationary, which is true for minimum phase systems comprising constant resistors, capacitors and inductors without drift. This relation is described by the Kramers-Kronig relations, which state that for causal complex plane spectral data there is a dependency between magnitude and phase. The real part of a spectrum can be obtained by an integration of the imaginary part and vice versa as described in the Kramers-Kronig equations. For impedance spectroscopy this has the consequence that we do not have to measure both magnitude and phase [7].

As we will see in the next section, the measurement of both magnitude and phase of an electric signal can be problematic to implement, especially if needed over many decades of frequencies. This paper proposes a new method to measure the amplitude of sinusoidal signals which is applicable for impedance measurements over a wide frequency span.

II. Conventional Amplitude Measurements

To measure an impedance at a single frequency, a sinusoidal wave function has to be imposed for the voltage, while the current is measured or vice versa. From the detected signal (and preferably for the applied stimulus signal as well), the amplitude and phase have to be detected. The measurement of phase and amplitude are two different problems. The measurement of phase is a timing problem which can normally be done with digital circuitry. The accuracy depends on the clock resolution and the jitter of the interfacing circuitry. The measurement of amplitude is an analog electronics problem. To measure the amplitude of a sine wave, the basal method is to use an envelope detector as shown in Fig. 1. The maximum voltage is copied to

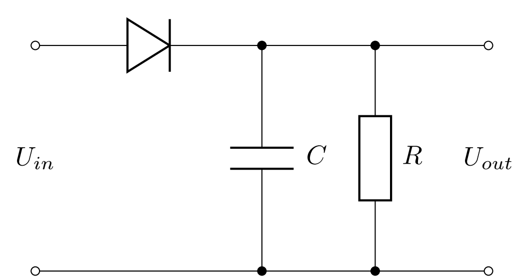

Fig. 1. Conventional amplitude detection using an envelope detector requires frequency-dependent optimization of a capacitor and a resistor

the capacitor using a diode. One problem with this elementary circuit is that there will always be a voltage drop of 0.7V over the diode. Another problem is that the R-C combination can never be optimized for a broad frequency range in a single design.

Equipment manufacturer Keysight distinguishes six types of impedance measurements based on the requirements and conditions [8]. One of the most versatile methods for a broad range of frequencies is the "auto-balanced bridge" set-up where a known sinusoidal voltage is imposed and the current is measured using an I-V converter. To subsequently detect the amplitude and phases of the signals, complex digital circuitry is used which is referred to as the "vector ratio detector section".

The front-end circuits to do envelope detection and phase detection are critical and hard to design when needed over a large frequency range. In product implementations where electronics is placed around a low clock-frequency microcontroller we would like to keep these front-ends as simple as possible.

III. The Proposed Method

A. *Principle*

Consider a sinusoidal signal as indicated in Fig. 2 having an amplitude of 1. Now take ten samples from this signal at random phases, meaning the sample intervals are not necessarily equidistant.

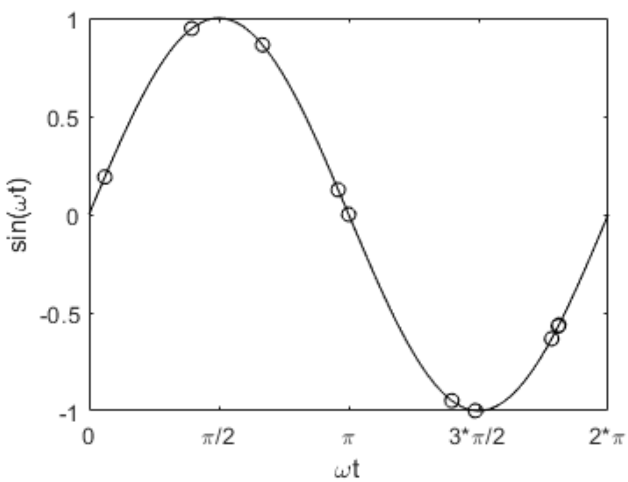

Fig. 2. A sine wave with some random samples

For the proposed method it does not matter whether these samples are from the same period: in fact, it is preferred to collect samples from multiple periods to ensure the full 0 to 2π phase range is uniformly sampled. The random phase sampling prevents the risk of sampling the same phase for all samples, so equidistant sampling using a sample frequency asynchronous to the signal frequency may suffice as well.

When the highest value is selected from the samples, the probability it is close to the real maximum will be high. A technical boundary condition is that the analog to digital converter has an instantaneous sample-and-hold circuit, which even low-cost converters have nowadays.

In practical implementations we have used an algorithm where the maximum and minimum value of the wave are detected from the same set of N samples. So, the complete envelope is detected without the need of extra samples, which has the additional benefit that the validity can be checked in case of signals symmetrical around zero. For the explanation and evaluation below, only detection of the maximum is taken.

So, our assumption is that the highest value of N samples, sampled under the conditions mentioned above, represents the maximum of the signal with a small error ε. The question rises how many samples N we have to take to conclude the error ε is within a confidence interval δ of the real maximum.

B. *Statistical analysis*

In Fig. 3 part of a cosine function is drawn around the maximum at t = 0. With random-phase sampling we try to find a sample that represents the maximum amplitude of the wave.

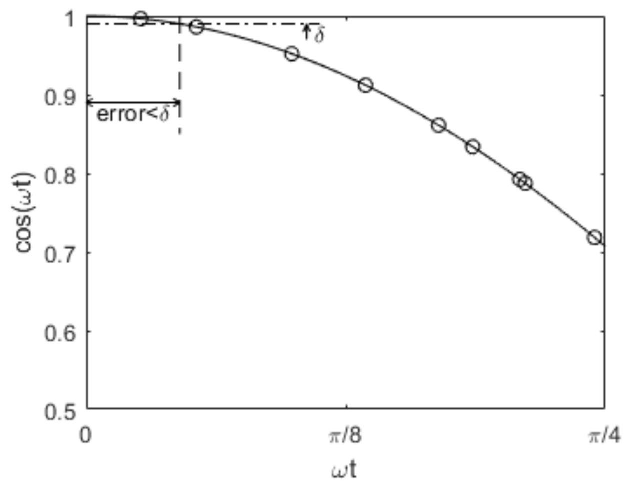

Fig. 3. The interval where a random sample lies within a distance δ of the maximum

We define a sample that is “close enough to the maximum” as a sample that deviates less than δ from the top, meaning it was taken from the interval between $\omega t_0 = 0$ and $\omega t_1 = \cos^{-1}(1-\delta)$. With this definition we consider only half of the cosine function because it is symmetrical around ωt = 0. Because we used random-phase sampling, all samples are uniformly distributed over the interval [0, π]. The probability a single sample is in the interval $x \in [\omega t_0, \omega t_1]$ where the error ε is smaller than δ is now given by

$$p\left(x\left(\delta\right) \mid \epsilon \leqslant \delta\right) = \frac{\omega t_1 - \omega t_0}{\pi} = \frac{\cos^{-1}\left(1-\delta\right)}{\pi} \quad . \tag{1}$$

Now we can calculate the probability at least one out of N samples is within the interval $x \in [\omega t_0, \omega t_1]$. The samples are again uniformly distributed, and independent. This probability is equal to the complementary chance all N samples are outside the interval $[\omega t_0, \omega t_1]$ expressed by

$$\begin{aligned} p\left(x\left(\delta\right) \mid \epsilon \leqslant \delta, N\right) &= 1 - p\left(x\left(\delta\right) \mid \epsilon > \delta, N\right) \\ &= 1 - \left(p\left(x\left(\delta\right) \mid \epsilon > \delta\right)\right)^N \end{aligned} \tag{2}$$

which results in

$$p\left(x\left(\delta\right) \mid \epsilon \leqslant \delta, N\right) = 1 - \left(1 - \frac{\cos^{-1}\left(1-\delta\right)}{\pi}\right)^N \quad . \tag{3}$$

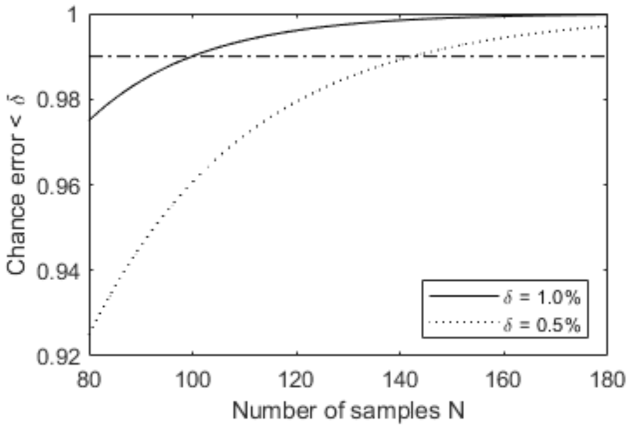

Fig. 6. The calculated probability the chosen maximum is closer than δ to the real maximum

This function is plotted in Fig. 4 for $\delta = 0.01$ and $\delta = 0.005$. From this graph we can see how many samples should be taken to conclude with a confidence of 1.0% (dash-dot line) the largest sample is closer than δ to the real maximum. This is N = 100 for $\delta = 0.01$ and N = 143 for $\delta = 0.005$.

C. Simulation

To get a better feeling for the number of samples needed to find an appropriate maximum a simulation is done. In MATLAB, a script was written which selects random N samples from a full period of a sine wave. The highest value is taken as the estimated maximum value. This script was executed 100000 times for each N so that the average detected values can be calculated. The results are in Fig. 5.

Now we can see that the confidence of finding the maximum as close as $\delta = 0.01$ is reached after N = 35 within a 2σ range. This simulation yields a lower number (N = 35) for the condition where 1% of the experiments give a larger error than $\delta = 0.01$ than the theoretical calculation (where N = 100).

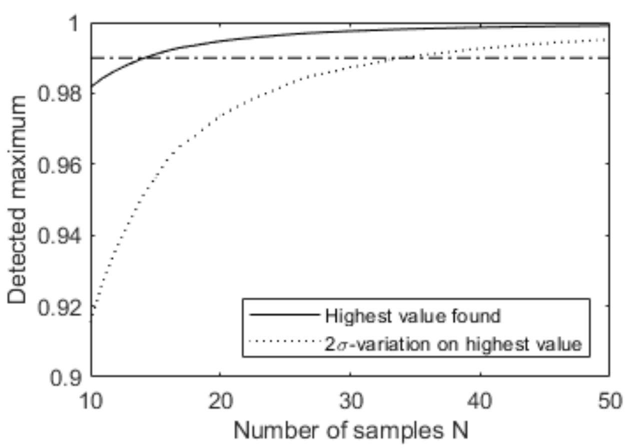

Fig. 5. Detected maximum from simulation

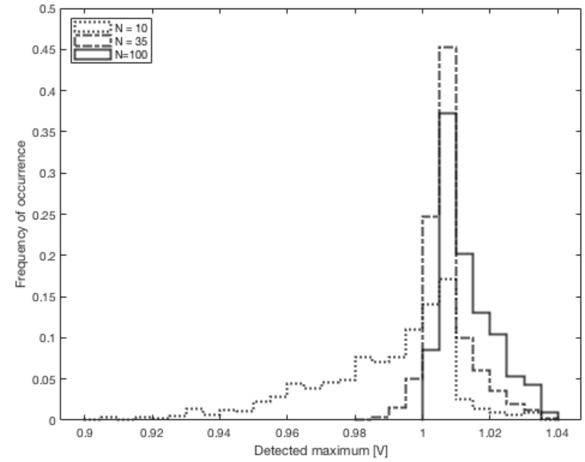

Fig. 4. Detected maximum values from a sine wave with amplitude 1V

IV. MEASUREMENTS

To verify whether the proposed method works, and to study the influence of the number of samples N further, some measurements were done.

A. Measurement setup

A sine wave of 2 kHz having an amplitude of 1 V was generated with a Tektronix AFG3021B function generator. On a Tektronix TBS2000 oscilloscope, the signal was verified to have a peak-peak value of 2.000 V (measurement resolution 25 mV). Sampling was done using a National Instruments PCI-6024E 12-bit data acquisition board, controlled and processed using MATLAB.

The sampling frequency was chosen at an odd value of 1234 Hz to avoid accidental synchronous sampling of the 2 kHz sine wave generator. This mimics the random-phase sampling in an easy way.

B. Measurements

Three series of experiments were done: maximum detection from N = 10, 35 and 100 samples respectively. For each N, 1000 maximum estimations were taken to be represented in three histograms having a bin-width of 5 mV. The bin-width of 5 mV corresponds to an error range of 0.5% because the expected maximum is 1 V. The three histograms are in Fig. 6. They are plotted normalized: the sum of all bars for a single plot is 1.

C. Discussion

First of all, the most likely maximum, being the maximum that was detected the most, is between 1.005 V and 1.010 V. This means that the method tends to detect the maximum. The fact that it is some millivolt above the real maximum of 1 V may have two causes: (1) The real maximum may not be 1.000 V, but slightly higher because of a measurement error in the TBS2000 oscilloscope. (2) We are detecting the maximum of sine wave plus additional noise – this method does not average out noise

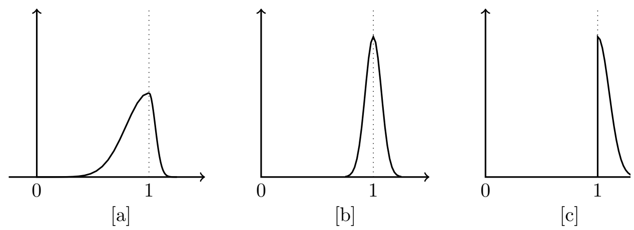

Fig. 7. Three different frequency of occurrence distributions functions for the detected maximum values. [a] N is too low, [b] N is correct, [c] N is unneccesarily high

in a single determination N. Both assumed errors are systematic, so turn up equally independent of N.

The number of samples N = 10 is obviously too low. A significant number of detections is below the 0.95 V. The histogram for N = 10 has a dominant tail down from the real maximum signal value. This shape is sketched in Fig. 7a: the real maximum of the sine wave is 1 in this sketch. Many estimations are too low.

Now we look back to the measurements in Fig. 6 for N = 35 samples. In that case, the result is closer to a normal distribution. It looks like a maximum is detected on top of which normally distributed signal noise is present. The characteristic shape is sketched in Fig. 7b. The chosen number of samples is high enough to find a maximum, but between experiments we observe the random signal noise.

Increasing from N = 35 to N = 100 samples the left tail of the normal distribution is copied on top of the right tail forming a half-normal distribution. This was seen in more experiments and is also visible in Fig. 6: the right-hand of the normally distributed bell shape is structurally above the one of lower values of N. The interpretation is that when going from N = 35 to N = 100 samples, the normal distributed noise on top of the sine wave signal becomes subject of the maximum detection as well. The characteristic half-normal distribution is sketched in Fig. 7c. This is a phenomenon that could not yet be predicted with the calculation of Fig. 4 and the simulation of Fig. 5 because these both did not have signal noise in the underlying model

V. CONCLUSION

Detection of the maximum of a sine wave shaped signal is successfully validated using a new method based on statistics. The method is suitable in impedance measurement applications where detection of the phase is not of primary interest. The advantage is that the signal amplitude can be detected over many decades of frequencies. Therefore, simple low rate sampling can be used, so the method can be implemented in simple microcontrollers. For sine waves, the theory predicts than N = 100 are enough samples to detect the maximum with a precision of 1%. Simulations and measurements show that taking less samples, for example N = 35, still gives a reliable result, and is even favorable because signal noise is still visible in the detected maximum.

In the presented results, sufficient randomization on the sampling was done to guarantee a sampling that is asynchronous with this signal. Future work must be done on the mathematical conditions behind this method. To be more specific, the relation between the sample taking and the expectation value of the estimated maximum is not yet understood. In addition, the relation between the optimum number of samples N and the signal noise was only optimized by simulations and not by an explicit expression.

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